Applied Virtual Reality in 3D Geometry

Konstantinos Sakkas*, Alexandra Tsogka*, Nikolaos Giannakeas*, Katerina D. Tzimourta*†, Alexandros T. Tzallas*, Euripidis Glavas*

* Department of Informatics and Telecommunication, University of Ioannina, Arta, Greece

†Dept. of Electrical & Computer Engineering, University of Western Macedonia, Kozani, Greece

ksakkas@uoi.gr, alexandratsogka01@gmail.com, giannakeas@uoi.gr, ktzimourta@uoi.gr, tzallas@uoi.gr, eglavas@uoi.gr

***Abstract*— The development of Virtual Reality technologies has improved people's daily lives in many sections. Nowadays, it is applied in fields such as Health, Work, Telecommuting and in a large number of other sectors. Education from different perspectives is something that constantly concerns researchers and virtual reality developers, which is why we come across a series of virtual reality applications for situation simulation and student education. Mathematics and especially geometry which is interrelated with the concept of the implementation of applications in three-dimensional space, as well as VR applications, from the perspective of learning are often considered complicated as there are three-dimensional objects that are not easily perceived only through a book. In this paper we examine how through virtual reality geometric concepts can be better understood but at the same time to examine geometric shapes from different angles and perspectives.**

***Keywords*— Virtual Reality, Geometry, Mathematic, Three-Dimensional Space, Conical Sections, Education, Learning.**

I. INTRODUCTION

Virtual Reality technology has been constantly developing in recent years in more and more fields of technology and education. By the term virtual reality, we are referring to a computer-generated environment with scenes and objects that appear to be real, making the user feel they are immersed in their surroundings. This environment is perceived through a virtual reality device which is called a VR Headset and such a device is displayed in Figure 1. Over the years, technology and virtual reality devices have taken a big rise and we now find VR applications in several areas of technology and learning.

In learning and teaching, virtual reality applications are used with the aim of understanding the subjects taught but also for interaction with students through a pleasant environment.

Fig. 1. Oculus Quest 2 [1]

We acknowledge the support of this work by the project "Immersive Virtual, Augmented and Mixed Reality Center Of Epirus" (MIS 5047221) which is implemented under the Action "Reinforcement of the Research and Innovation Infrastructure", funded by the Operational Programme "Competitiveness, Entrepreneurship and Innovation" (NSRF 2014-2020) and co-financed by Greece and the European Union (European Regional Development Fund).

Traditional methods of education have several limitations and difficulties, as a result of which teaching becomes meaningless. It had already been turned out during the COVID-19 pandemic, where imparting the knowledge and especially subjects that required practical training, through electronic platforms, seemed to be almost impossible [2].

However, through a VR device and a combination of methods, the training aim could be both easily perceptive and fun at the same time as:

- Teachers can prepare for their lesson through a virtual classroom.
- Students can comprehend concepts and practice in a safe and comfortable learning environment.
- Teachers are given the ability to analyze a three-dimensional object associated with a concept, from different perspectives, in real time.
- Trainees can remember and become familiar with new knowledge in their personal space [3].

A term that encompasses the technology of Virtual Reality and related technologies, classifies them as Reality Enhancing Technologies (RET) because their goal is to modify or improve the reality experienced by a user by introducing a virtual environment with virtual three-dimensional objects. In fact, since 2020, RET has been placing restrictions on the appropriate material and technological illiteracy, thus concluding that teachers should have the appropriate qualifications to use RET technology [4]. Mathematics as well as geometry are subjects with complex concepts of equations and shapes and that is why RET technologies would certainly be effective in learning these concepts.

Fig. 2. Geometric Shapes

Geometric three-dimensional shapes are part of geometry, which is a branch of mathematics that studies sizes, shapes, angles of positions as well as dimensions. Flat shapes such as squares, circles, and triangles are called 2D shapes and involve only two dimensions.

979-8-3503-9858-8/22/$31.00 ©2022 IEEE

Solid objects or otherwise three-dimensional objects have a third dimension to determine height or depth. However, this is a problem that concerns teachers much more as the depiction of a three-dimensional object on the board or on paper is almost impossible, while even with the use of a computer the comprehension results are ranked quite low [5]. Geometric equations often produce intricate three-dimensional objects that are quite difficult to imagine especially when the geometric object:

- Is depicted in a space that does not rotate in three dimensions, like a book.
- Is depicted only from one facet (e.g. frontside).
- It results from incisions of objects.
- It tends to infinity, such as lines or layers.

In the majority of these problems a three-dimensional representation will help to perceive shapes but as we move on to combinations of objects and object intersections the problem remains [6]. Through a Literature Review, it was observed that the majority of papers studied have as their object of study both the applications of VR and AR, however, the majority of the applications that have been implemented are in AR. Something that was observed through the study of similar papers is that the verification of the correctness of the application was carried out with questionnaires regarding the participants. The majority of the surveys were conducted on pupils and / or students but also on professors.

From literature review we observe methods and results from similar articles they researched in terms of Virtual Reality, geometry, mathematics and intersections of geometric objects [7]–[15].

Through the literature review, it was observed that there is a small amount of articles that have implemented integrated virtual reality applications for learning or even easier understanding of geometric equations and shapes that arise through them or even something that often puzzles students such as understanding the intersections that arise between shapes. In this article we propose the use of virtual reality to understand the geometric shapes that come up through geometric equations as well as the conical intersections between shapes or produced by a combination of equations.

II. Methodology

A. 3D Geometry and 3D Shapes

With the term three-dimensional geometry we refer to a method where its goal is to represent a point, a line or a plane in relation to the axes x, y and z as displayed in figure 3. The coordinates of any point in three-dimensional geometry have three coordinates (x, y, z). In the three-dimensional Cartesian coordinate system there are the three axes x,y,z which are mutually perpendicular to each other and have the same units of length [16].

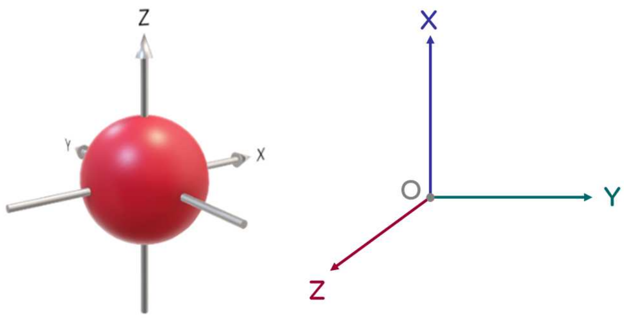

Fig. 3. Three-dimensional Space

The intersection point of these three axes is the principle O as shown in figure 3 and they divide the space into eight octane:

$$(x,y,z)\begin{cases}(+x,+y,+z)\\(-x,+y,+z)\\(+x,+y,-z)\\(-x,+y,-z)\\(+x,-y,+z)\\(-x,-y,+z)\\(+x,-y,-z)\\(-x,-y,-z)\end{cases}$$

In three-dimensional geometry there are important concepts that were examined in our methodology, such as the directional ratio, the direction cosine, the distance formula, the mean point formula and the intersection formula.

Direction Ratios

A point A (a, b, c) is represented as a vector with a vector position:

$$\vec{O}A = a\vec{\imath} + b\vec{\jmath} + c\vec{k}$$

This ratio represents the vector line with reference to the axes x, y, z.

Direction Cosine

The directional cosine gives the relationship of a vector or a line in a three-dimensional space, with each of the three axes. Directional cosines for a vector.

$$\vec{A} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$$

$$Cosa = \frac{a}{\sqrt{(a^2+b^2+c^2)}}$$

$$Cosb = \frac{b}{\sqrt{(a^2+b^2+c^2)}}$$

$$Cosc = \frac{c}{\sqrt{(a^2+b^2+c^2)}}$$

Directional cosines are also represented by l, m, n, and we can prove that:

$$l^2 + m^2 + n^2 = 1$$

Distance Formula

The distance between two points (x1, y1, z1) and (x2, y2, z2) is the smallest distance and equals the square root of the sum of the square of the difference in coordinates x, y and z of the two points at a time.

$$D = \sqrt{(x2 - x1)^2 + (y2 - y1)^2 + (z2 - z1)^2}$$

Mid-Point Formula

The middle is located on the straight line that connects the two points and is located directly between the two points.

$$(x, y) = \left(\frac{x1 + x2}{2}, \frac{y1 + y2}{2}, \frac{z1 + z2}{2}\right)$$

Section Formula

The intersection formula is useful for finding the coordinates of a point that divides the straight segment that joins the points (x1, y1, z1) and (x2, y2, z2)in the ratio M : n [17]

$$(x, y) = \left(\frac{mx2 + nx1}{m + n}, \frac{my2 + ny1}{m + n}, \frac{mz2 + nz1}{m + n}\right)$$

B. 3D Geometry Education and Virtual Reality

In learning it is now customary to use Information and Communication Technologies known as ICT brevity. In geometry we already find several ICT applications such as logo, Cabri-Geometre and Sketchpad, however the main disadvantage of these technologies is that they are operable in 2D environments [18].

A solution to this problem is provided by virtual reality where a widespread problem is solved with traditional teaching methods. Studies have proven that virtual reality leads to increased student participation in activities by providing an opportunity to stimulate student engagement, an opportunity to address boring or low-impact lessons or modules.

Through the practical, interactive and immersive experience, the way of classical learning changes, thus offering new powerful experiences. In this way, the comprehension of even complex geometric equations can be made to a greater extent understood [19].

Another feature in learning is that virtual reality provides the opportunity for constructivist learning, i.e. students are given the opportunity to construct their own knowledge based on their experiences, therefore students can solve problems but also collaborate.

One fact often mentioned by students is that they cannot relate what is taught in the classroom to the real world, and here the solution is provided even in geometric intersections where through virtual reality we can understand the concept of intersection in real environments.

The ability to visualize brings the greatest advantage in the fields of mathematics and geometry as Virtual Reality enables students to create any difficult concept they come into contact with and view from any angle, three-dimensionally and in 360o, the object they want - something similar is shown figure 4 [20].

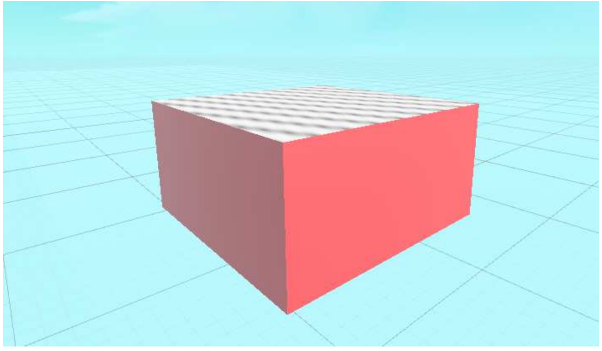

Fig. 4. Incision of a three-dimensional cube

C. Experimental procedure

In the experimental process participated 20 people, 8 women and 12 men, with an average age 25. The experiment was carried out in an equipped space with speakers and without obstacles. The experimental process began by presenting to the participants the technology of Virtual Reality and relevant details on how virtual reality devices work.

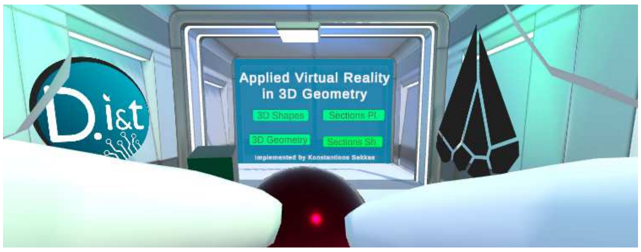

Fig. 5. Customization Environment and Application Menu

Then the participants were asked to wear the virtual reality device and with appropriate instructions they found out the safety of its use and movement with the corresponding device. Immediately afterwards, the participants had a point in time at which througout the experiment they could get accustomed to the device, the space and the controls as well as learning hand co-ordination. In the next part participants were asked to choose one of the three stages of the application:

- Navigating three-dimensional shapes stemming from geometric equations
- Introduction to three-dimensional geometry
  - Proportions of direction and cosine
  - Distance and meeting formula in three-dimensional form
- Intersections of three-dimensional shapes with the plane relative to different x,y,z
- Intersections of three-dimensional shapes with other three-dimensional shapes in relation to different x,y,z.

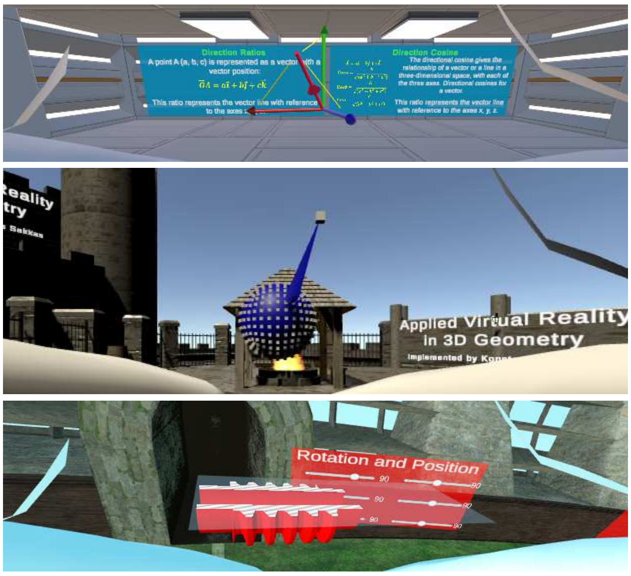

Fig. 6. Experimental Rooms

After the experiment process, participants were asked to evaluate the experiment through a questionnaire, in terms of their experience with virtual reality, the knowledge with which they came into contact and the evaluation of learning through virtual reality.

III. Results and Discussion

After the experimental procedure each participant was asked to evaluate the experiment through a questionnaire. The questionnaire was divided into two parts, the first was about whether the participants know the concept of 3D Geometry, if they have been in contact with a 3D Space as well as if they know the Cartesian Coordinate System, this was the category of general questions, then there were and special questions on specific geometric concepts such as:

- Direction Ratios
- Direction Cosine
- Distance Formula
- Mid-Point Formula
- Section Formula

The last but also quite important question was whether the participants were related to the subject of three-dimensional geometry.

The second part of the questionnaire was about the concept of Virtual Reality where the participants should initially answer whether they know what virtual reality is. Then the questions related to the combination of Virtual Reality and Learning with questions such as:

- How useful do you think Virtual Reality (VR) is in Mathematics and Geometry Education?
- Have you used a VR app related to Geometry or Maths
- Would you choose conventional training or VR training?

The results were very positive for the experimental process as the majority of the participants stated that they were completely satisfied with the whole process and that the understanding of the concepts increased by a large difference compared to the traditional method.

IV. Conclusions

Summarizing the conclusion that comes from the research carried out is that in education and especially in difficult areas such as Mathematics and Geometry, traditional techniques are not sufficient for understanding complex concepts. In comparison, the participants seeing in the presentation the concepts implemented in the Virtual World stated from the beginning that they are quite difficult to understand even in their imagination, however after the experimental process the participants stated that they can now to a greater extent understand these concepts as well as that Virtual Reality should be integrated into more concepts because through it the understanding of complex concepts can become easier.

In future extensions, it will be examined how the specific experimental process will be able to penetrate more complex concepts as well as how a mathematical system could be solved in a virtual environment and this solution graphically illustrated.

References

[1] "Oculus Quest 2, NC STATE University Libraries"

[2] V. Kumar, S. Gulati, B. Deka, and H. Sarma, "Teaching and Learning Crystal structures through Virtual Reality based systems," Advanced Engineering Informatics, vol. 50, p. 101362, Oct. 2021, doi: 10.1016/j.aei.2021.101362.

[3] K. Li and S. Wang, "Development and application of VR course resources based on embedded system in open education," Microprocessors and Microsystems, vol. 83, p. 103989, Jun. 2021, doi: 10.1016/j.micpro.2021.103989.

[4] D. A. Buentello-Montoya, M. G. Lomelí-Plascencia, and L. M. Medina-Herrera, "The role of reality enhancing technologies in teaching and learning of mathematics," Computers & Electrical Engineering, vol. 94, p. 107287, Sep. 2021, doi: 10.1016/j.compeleceng.2021.107287.

[5] V. Izard, P. Pica, and E. S. Spelke, "Visual foundations of Euclidean geometry," Cognitive Psychology, vol. 136, p. 101494, Aug. 2022, doi: 10.1016/j.cogpsych.2022.101494.

[6] A. Kovarova and M. Sokolsky, "Using virtual reality for teaching solid geometry: A case study for a cube section," in 2011 14th International Conference on Interactive Collaborative Learning, Sep. 2011, pp. 428–433. doi: 10.1109/ICL.2011.6059620.

[7] W. Widada, D. Herawaty, K. U. Z. Nugroho, and A. F. D. Anggoro, "Augmented Reality assisted by GeoGebra 3-D for geometry learning," Journal of Physics: Conference Series, vol. 1731, no. 1, p. 012034, Jan. 2021, doi: 10.1088/1742-6596/1731/1/012034.

[8] Sudirman, P. Yaniawati, Mellawaty, and R. Indrawan, "Augmented reality application: What are the constraints and perceptions of the students during the covid 19 pendemic's 3D geometry learning process?," Journal of Physics: Conference Series, vol. 1783, no. 1, p. 012007, Feb. 2021, doi: 10.1088/1742-6596/1783/1/012007.

[9] T. F. Brustad, "Preliminary Studies on Transition Curve Geometry: Reality and Virtual Reality," Emerging Science Journal, vol. 4, no. 1, pp. 1–10, Feb. 2020, doi: 10.28991/esj-2020-01204.

[10] P. Salinas and R. Pulido, "Understanding the Conics through Augmented Reality," EURASIA Journal of Mathematics, Science and

Technology Education, vol. 13, no. 2, Dec. 2016, doi: 10.12973/eurasia.2017.00620a.

[11] Malamos, Athanasios & Mamakis, Georgios & Sympa, Paraskevi & Kotanitsi, Eleni & Crespo, Alfredo & Lopez, Alvaro, “Technical aspects in using X3D in virtual reality mathematics education (EViE-m platform),” pp. 284–291, 2008.

[12] K. Moustakas, G. Nikolakis, D. Tzovaras, and M. G. Strintzis, “A geometry education haptic VR application based on a new virtual hand representation,” in IEEE Proceedings. VR 2005. Virtual Reality, 2005., pp. 249–252. doi: 10.1109/VR.2005.1492782.

[13] T.-L. Sun, “Conics-enhanced vision approach for easy and low-cost 3D tracking,” Pattern Recognition, vol. 37, no. 7, pp. 1441–1450, Jul. 2004, doi: 10.1016/j.patcog.2004.01.009.

[14] H. Kaufmann and D. Schmalstieg, “Designing Immersive Virtual Reality for Geometry Education,” in IEEE Virtual Reality Conference (VR 2006), pp. 51–58. doi: 10.1109/VR.2006.48.

[15] H. Kaufmann, “Construct3D,” in Proceedings of the tenth ACM international conference on Multimedia - MULTIMEDIA ’02, 2002, p. 656. doi: 10.1145/641007.641140.

[16] Y. S. Kusumah, D. Kustiawati, and T. Herman, “The Effect of GeoGebra in Three-Dimensional Geometry Learning on Students’ Mathematical Communication Ability,” International Journal of Instruction, vol. 13, no. 2, pp. 895–908, Apr. 2020, doi: 10.29333/iji.2020.13260a.

[17] “3D Geometry.” https://www.cuemath.com/geometry/3d-geometry/ (accessed Aug. 29, 2022).

[18] A. Yeh, “VRMath,” in Extended abstracts of the 2004 conference on Human factors and computing systems - CHI ’04, 2004, p. 1061. doi: 10.1145/985921.985979.

[19] C. Christou, “Virtual Reality in Education,” in Affective, Interactive and Cognitive Methods for E-Learning Design, IGI Global, pp. 228–243. doi: 10.4018/978-1-60566-940-3.ch012.

[20] E. Hu Au and J. J. Lee, “Virtual reality in education: a tool for learning in the experience age,” International Journal of Innovation in Education, vol. 4, no. 4, p. 215, 2017, doi: 10.1504/IJIIE.2017.10012691..